Penalized B-Spline Regression to Analyze Trends in Reported Foodborne Illness

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INTRODUCTION

- Objective: Analyze temporal patterns in reported U.S. foodborne illness without specifying model form
- Illustrative Case: Salmonella
  - Annual and seasonal patterns
  - All Salmonella serotypes, principal serotypes

DATA

- FoodNet (Foodborne Diseases Active Surveillance Network)
- Reported illness counts by site, year, and month
- Population size by site and year
- Site composition stable since 2004

- All Salmonella Serotypes
- Principal Salmonella Serotypes
  - Typhimurium
  - Enteritidis
  - Newport
  - Account for over 40% of serotyped strains

METHODS

- Penalized B-spline Regression
  - Semi-parametric method – no assumed trend model form
  - B-spline basis functions provide local control, local fit is insensitive to points far removed
  - Penalized form of B-spline regression is insensitive to number, placement of join-points (“knots”)
- Wide range of applications

METHODS

- Generalized Additive Model for Poisson Regression
- $\text{Log}(E[y_i]) = \log(\text{population}_i)+\beta_0+f(\text{year}_i)+\varepsilon_i$
  - Smooth $f(\text{year}_i) = \sum B_k(\text{year}_i)\beta_k$
    - $B_k(x)$ = B-spline basis function
  - Year (nx1 vector) $\rightarrow$ **X** (nxk matrix)
    - Fit the model with basis functions as covariates
- $\text{Log}(E[y_{ij}]) = \log(\text{population}_{ij})+\beta_0+f_1(\text{year}_i)+f_2(\text{month}_j)+\varepsilon_{ij}$

METHODS

- At any given point, q+1 B-splines are non-zero (local control)
  - q = B-spline degree (e.g., q=3 for cubic)
  - B-splines sum to 1
- Basis dimension (k) = q + n’ (unconstrained)
  - n’ = no. intervals along domain
  - e.g., 2 internal knots divides domain into n’ = 3 intervals
- Eilers and Marx (1996) provides recursive algorithm for B-spline basis functions for uniformly spaced knots
- In practice, need to impose identifiability constraint → k-1 orthogonal columns (QR decomposition)
- Smoothness controlled by penalty term, fit insensitive to basis dimension

METHODS

- P-IRLS to obtain GLM likelihood maximization, s.t. smooth
- Given λ, min: $\left\|\sqrt{W}(z - X\beta)\right\|^2 + \lambda\beta^T S\beta$
  - λ = curvature penalty parameter
  - $w_i \propto [V(\mu_i)g'(\mu_i)^2]^{-1}$
  - $V[y_i] = \phi\mu_i$ (Generalized Poisson)
  - $z_i = g'(\mu_i)(y_i - \mu_i) + X_i\beta$
  - X = (constrained) design matrix
  - g = link function (log)
  - S (penalty matrix) = $D^T D$
    - penalize differences among neighboring β coefficients
  - For D = second order difference matrix ~ $\int[f''(x)]^2 dx$
    - measure of total curvature

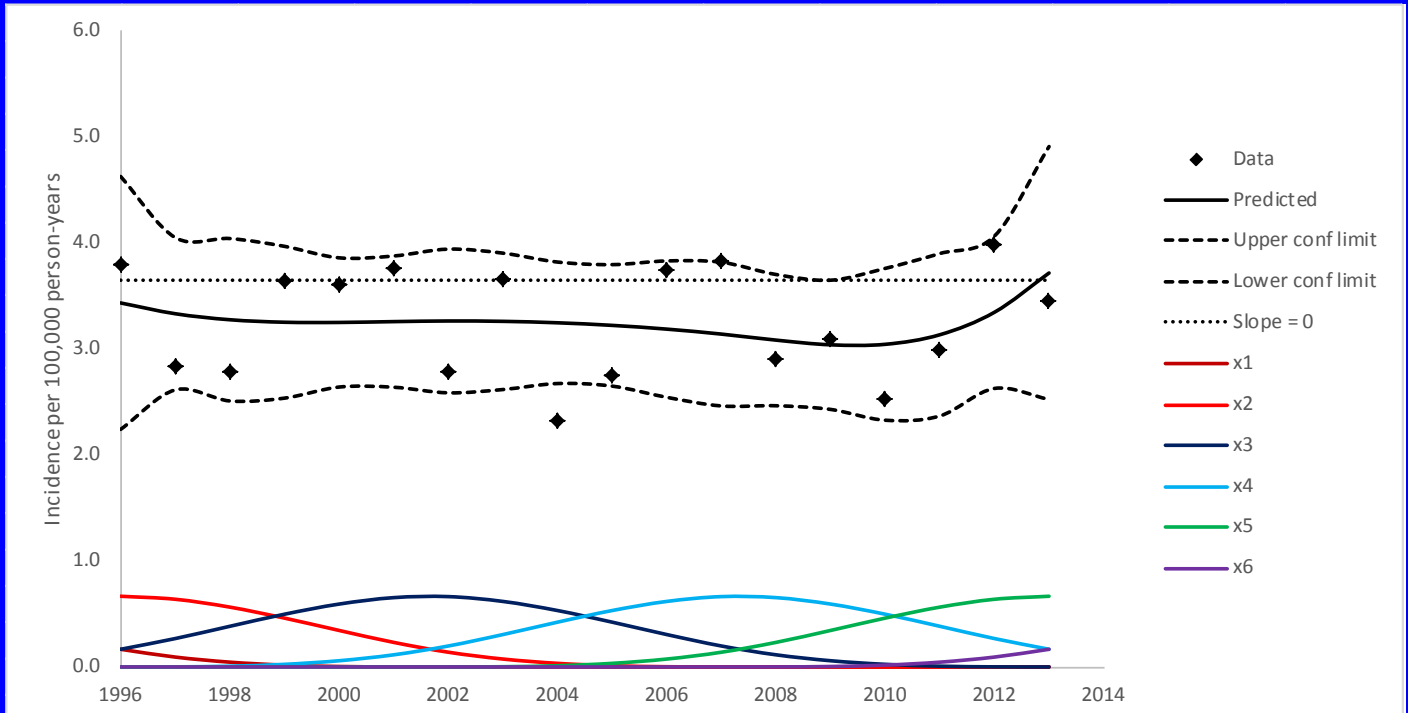
METHODS

- Select degree of smoothness (λ) based on model selection criterion (GCV)
- Effective degrees of freedom (edf) = tr(A)
  - where $\hat{\mu} = Ay$
- With λ = 0, tr(A) = k-1
- As λ→∞, GAM → Log-Linear Model (X →1 edf)

METHODS

- Uniform cubic B-spline basis with 2 internal knots
  - k = q(3)+ n'(3) = 6 unconstrained basis functions
- S (penalty matrix): 2nd order difference matrix
- All Sites
  - Composition of FoodNet sites stable since 2004
- Original 5 Sites
  - Attempt to control for changes in FoodNet composition over time

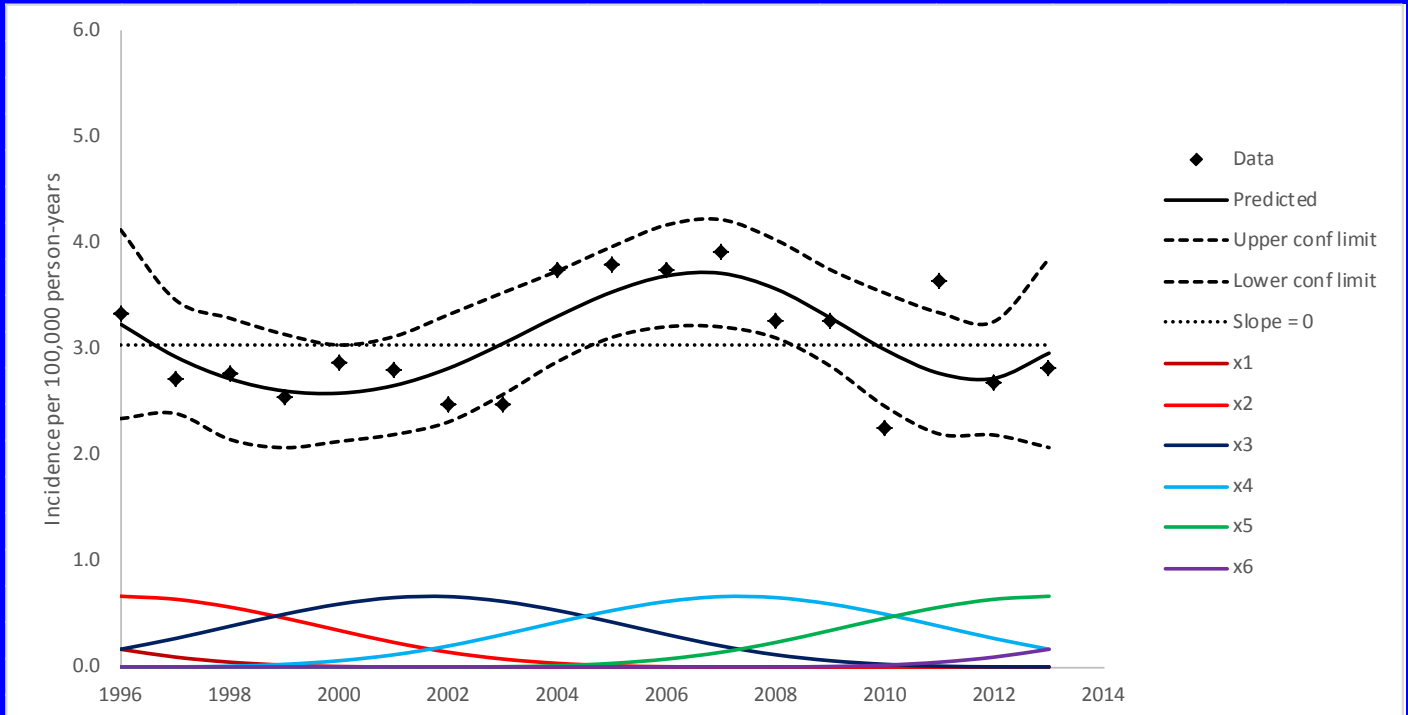
METHODS
B-Spline Provides Flexibility



	PENALTY	0.00										
	AICC	1.351276										

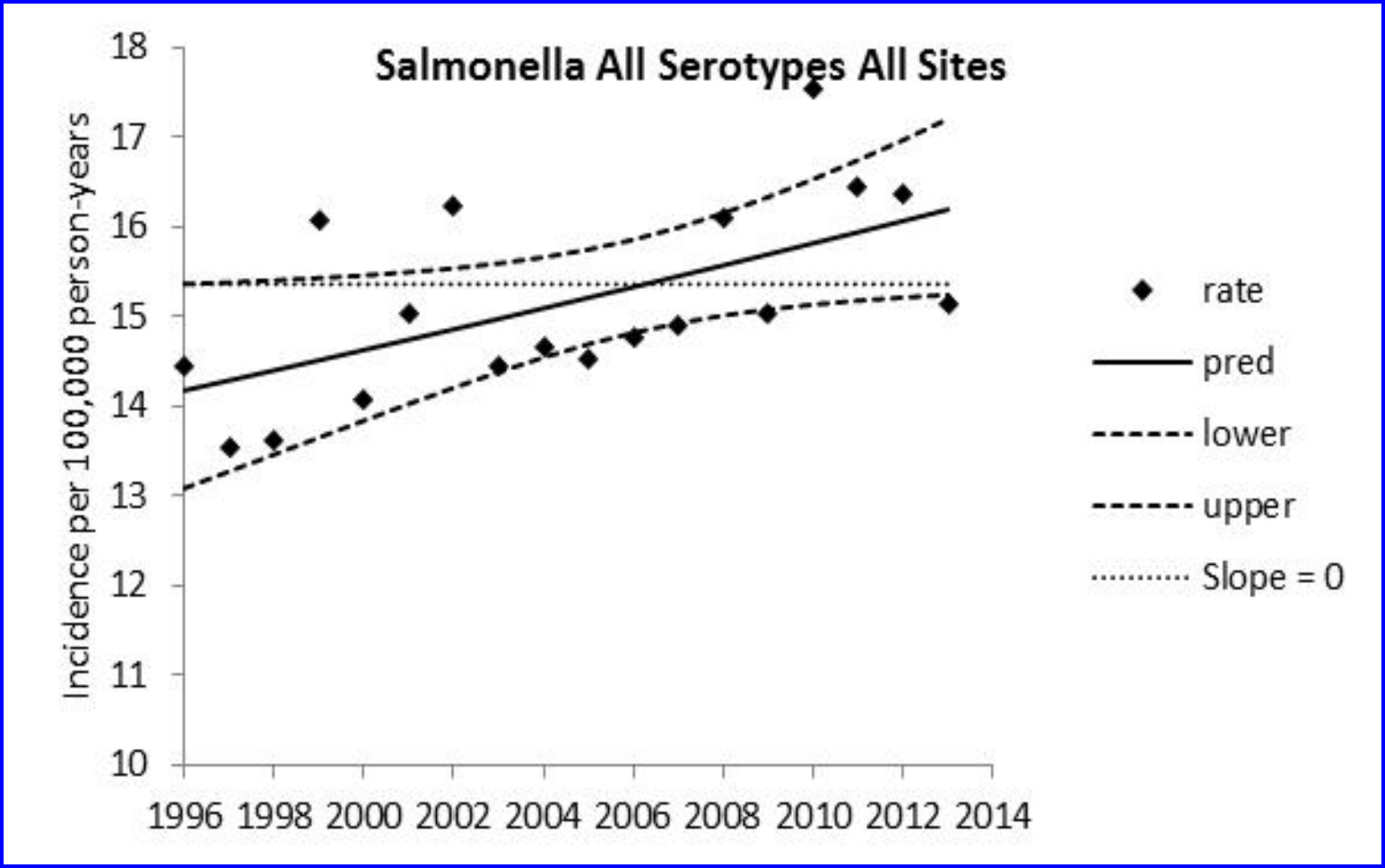
METHODS

Penalized B-Spline Avoids Overfitting



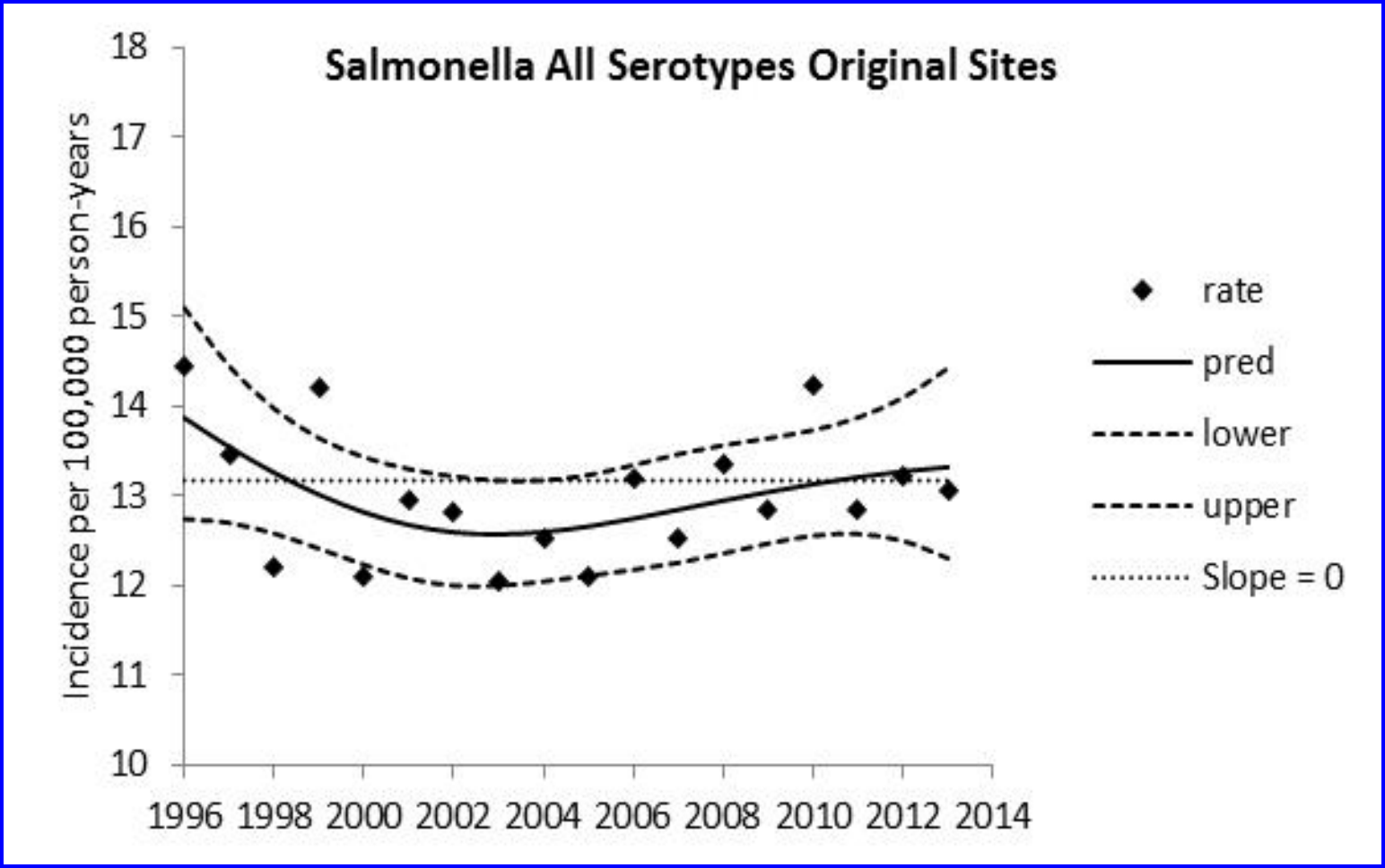
	PENALTY	0.00
	AICC	0.766499

RESULTS



Powell: P-Spline Regression

RESULTS



Powell: P-Spline Regression

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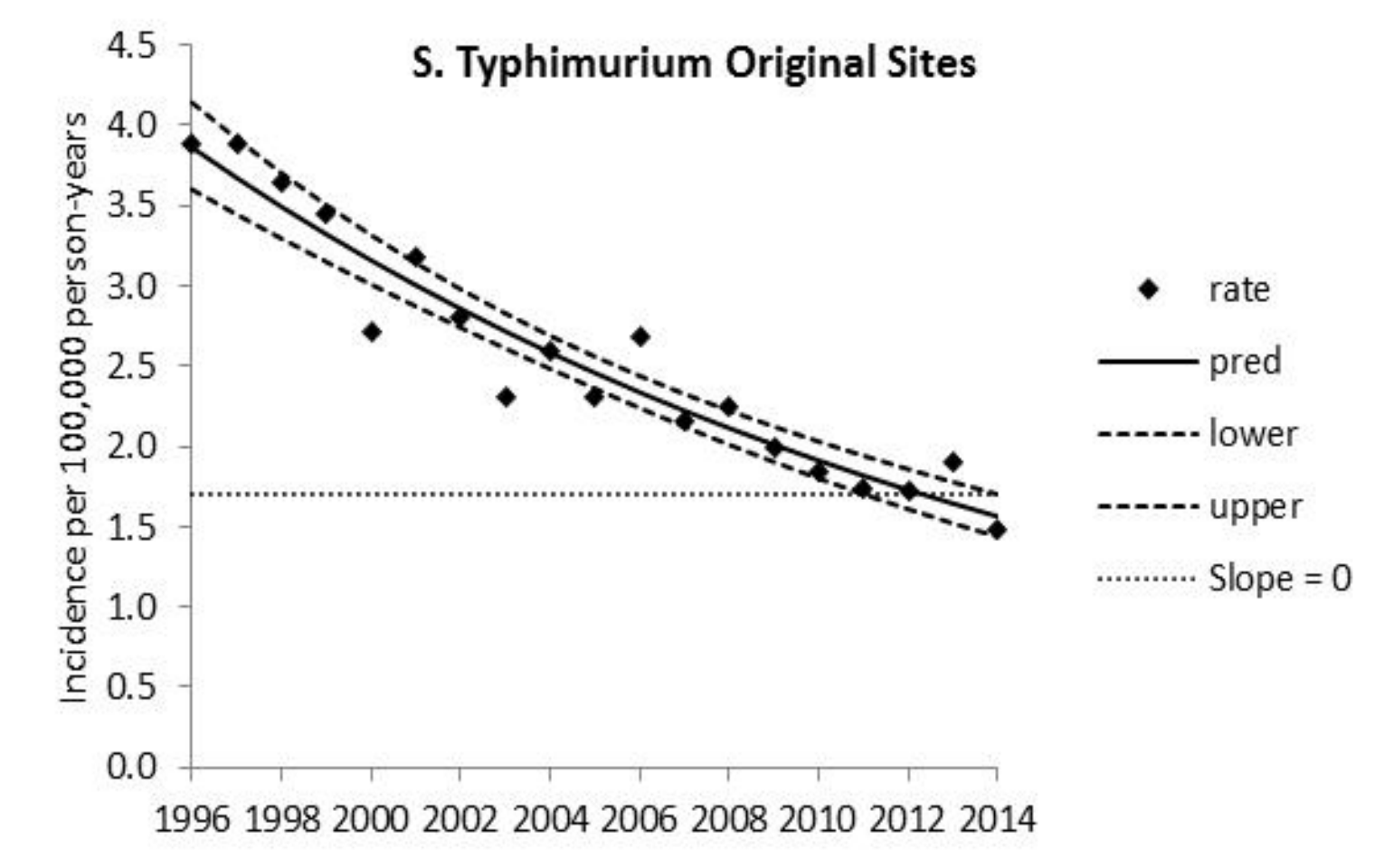

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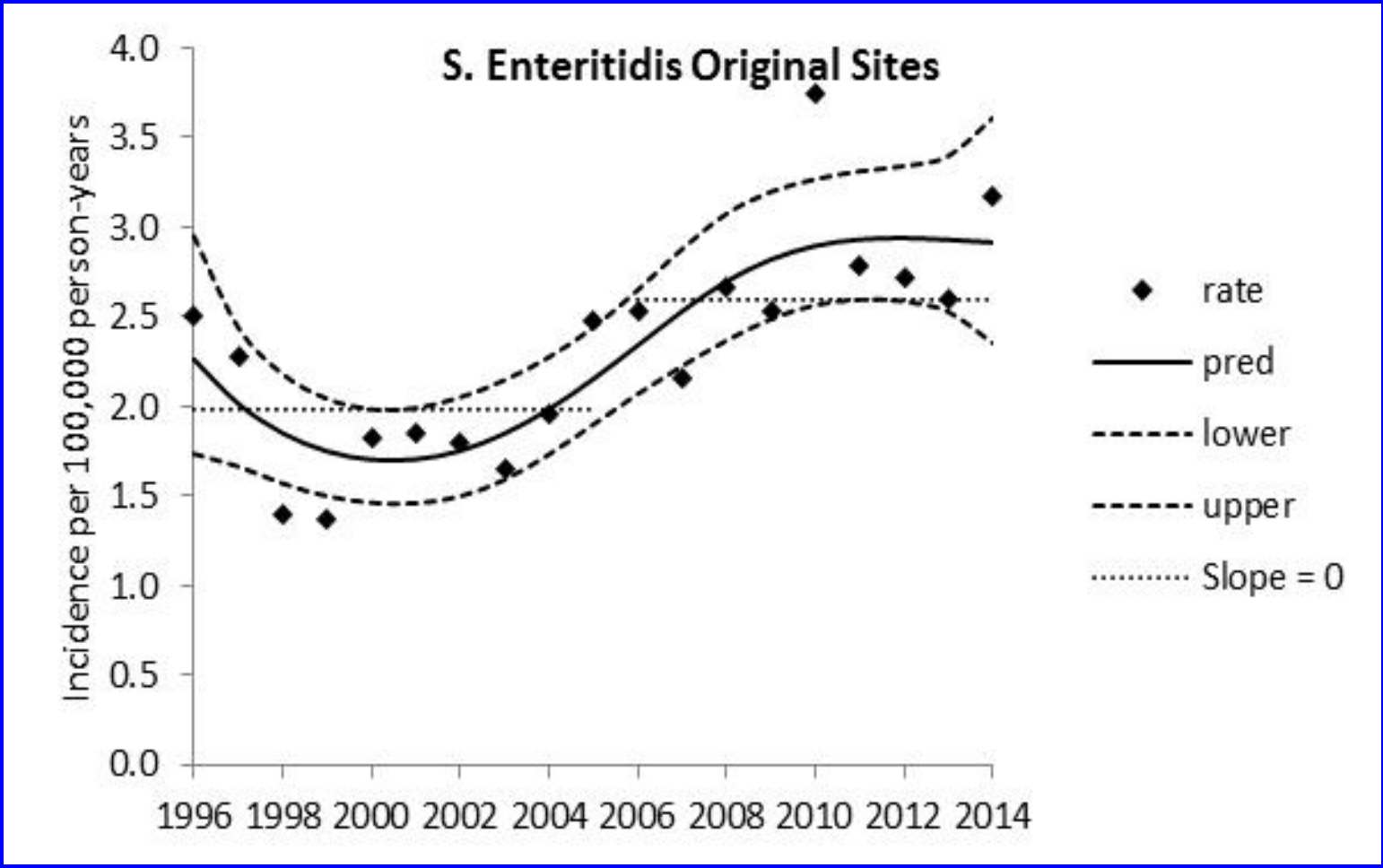

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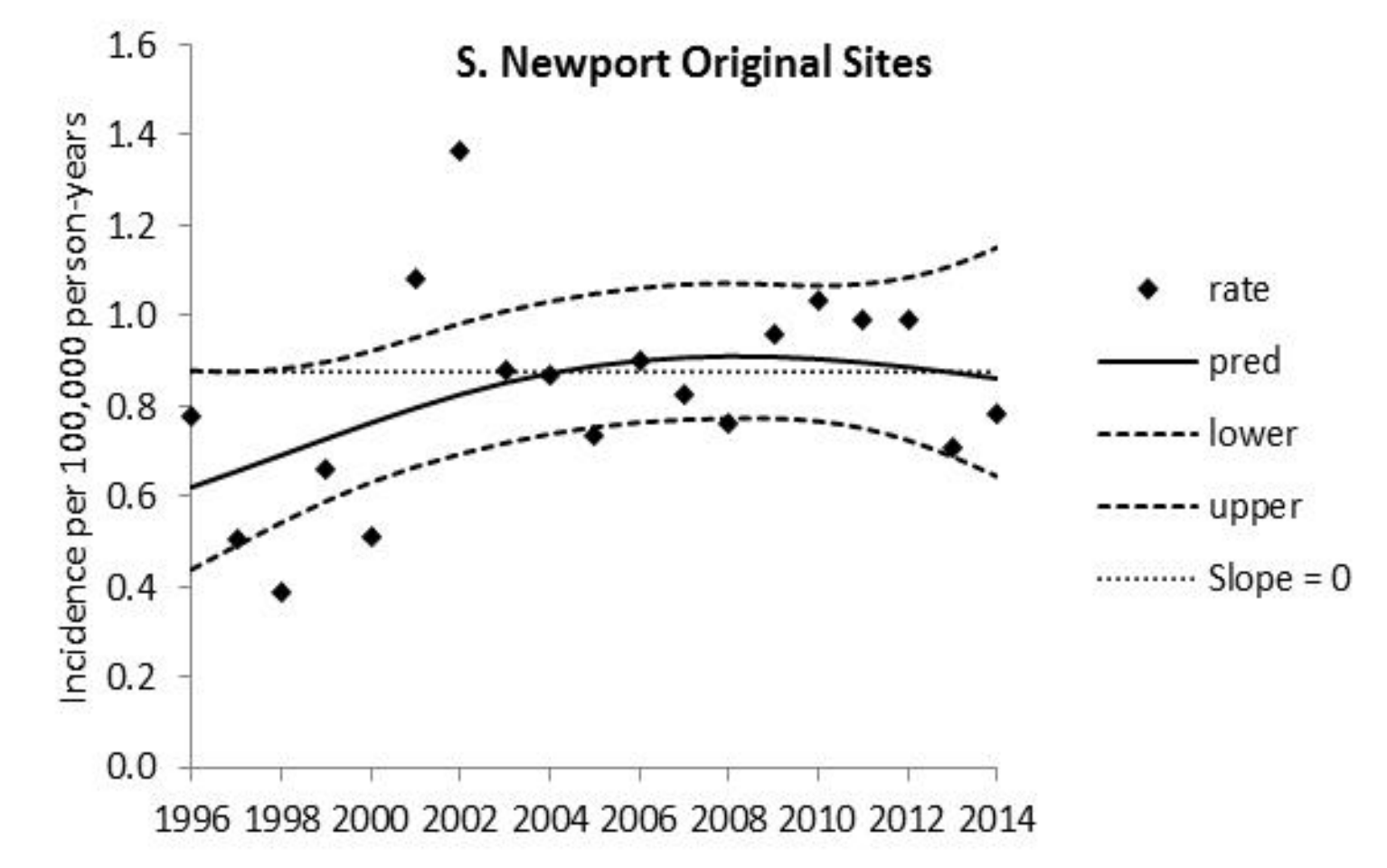

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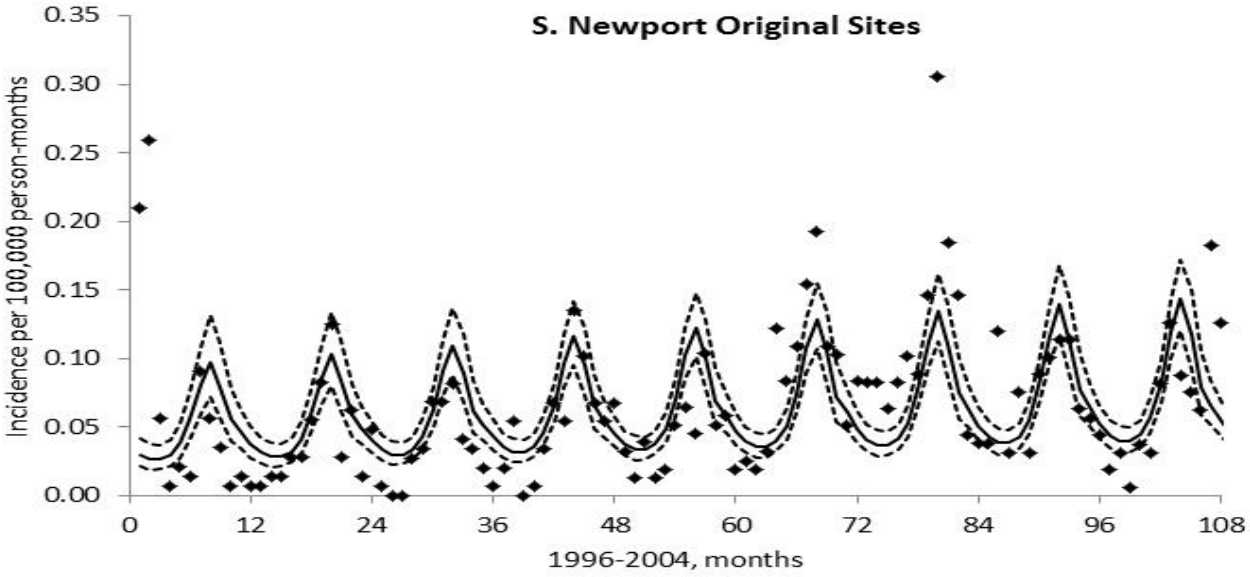
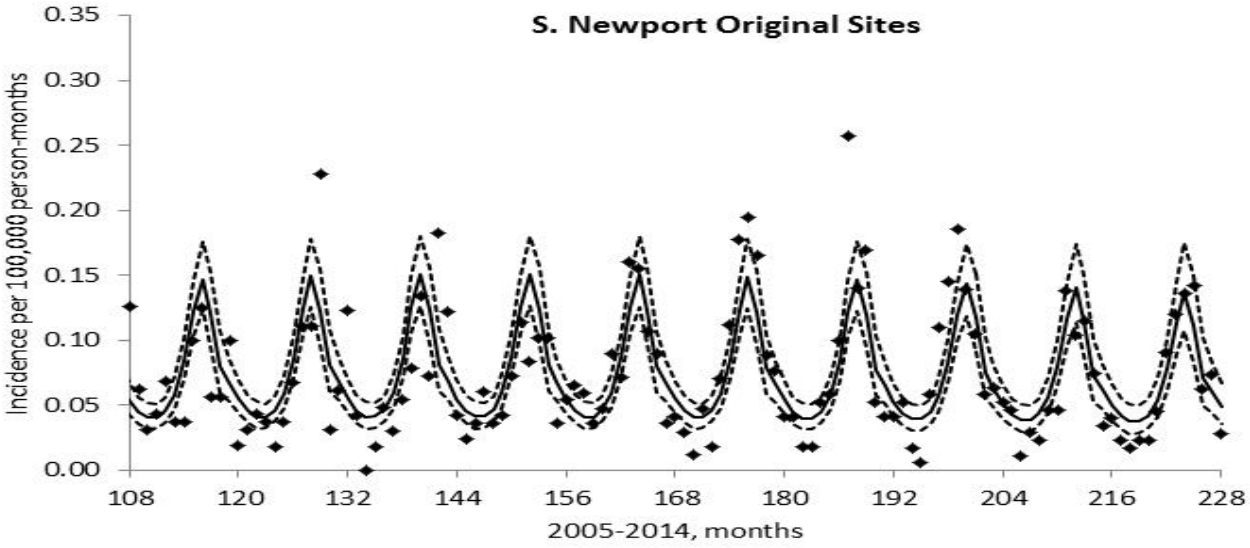
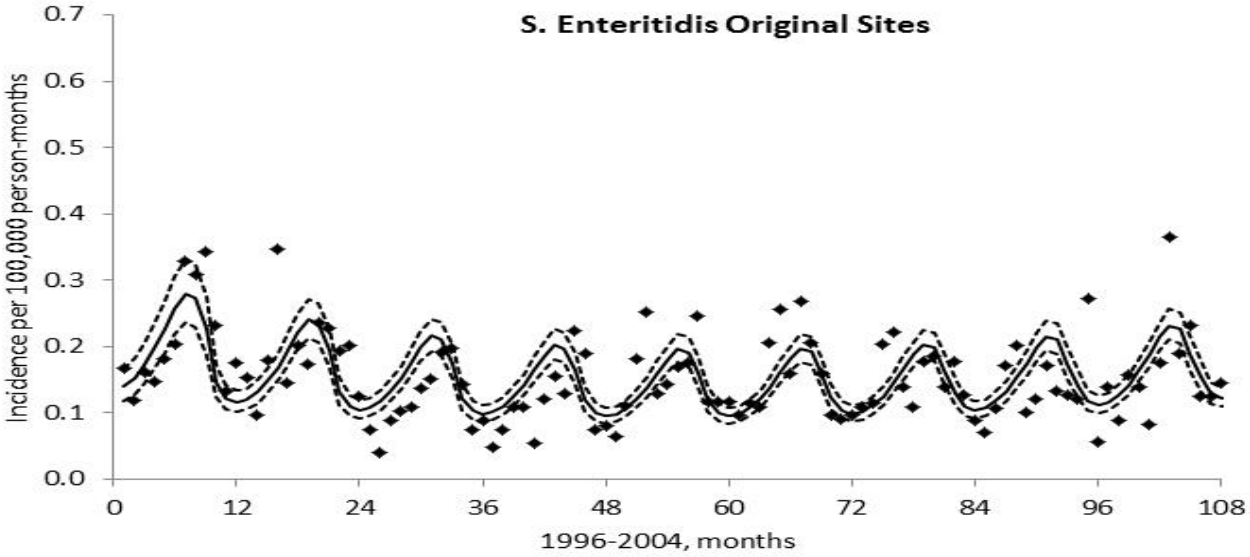
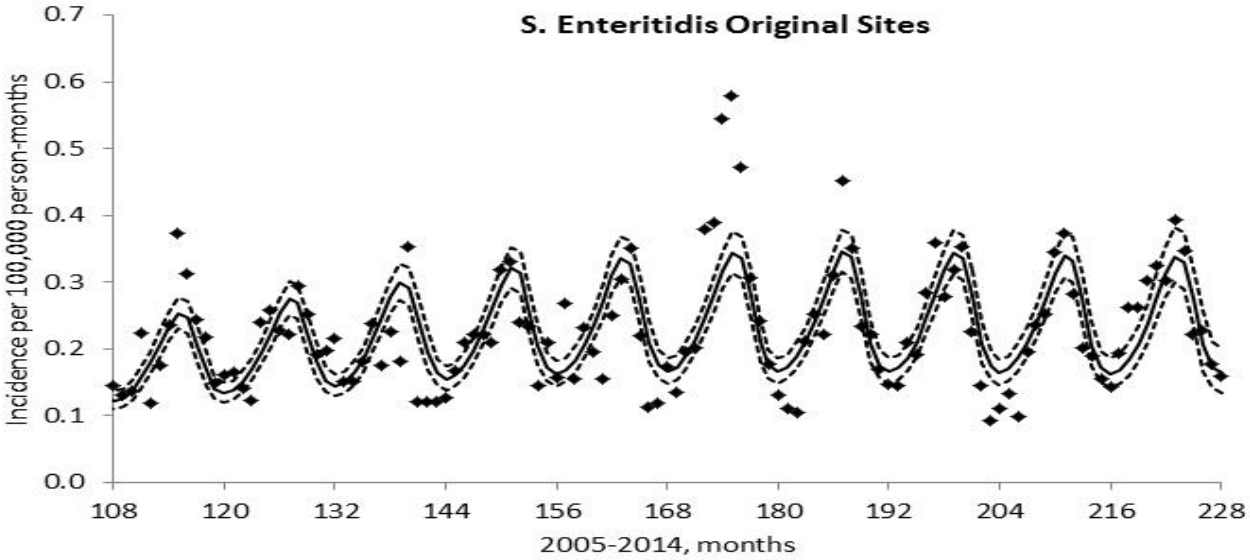




Powell: P-Spline Regression

RESULTS





Powell: P-Spline Regression

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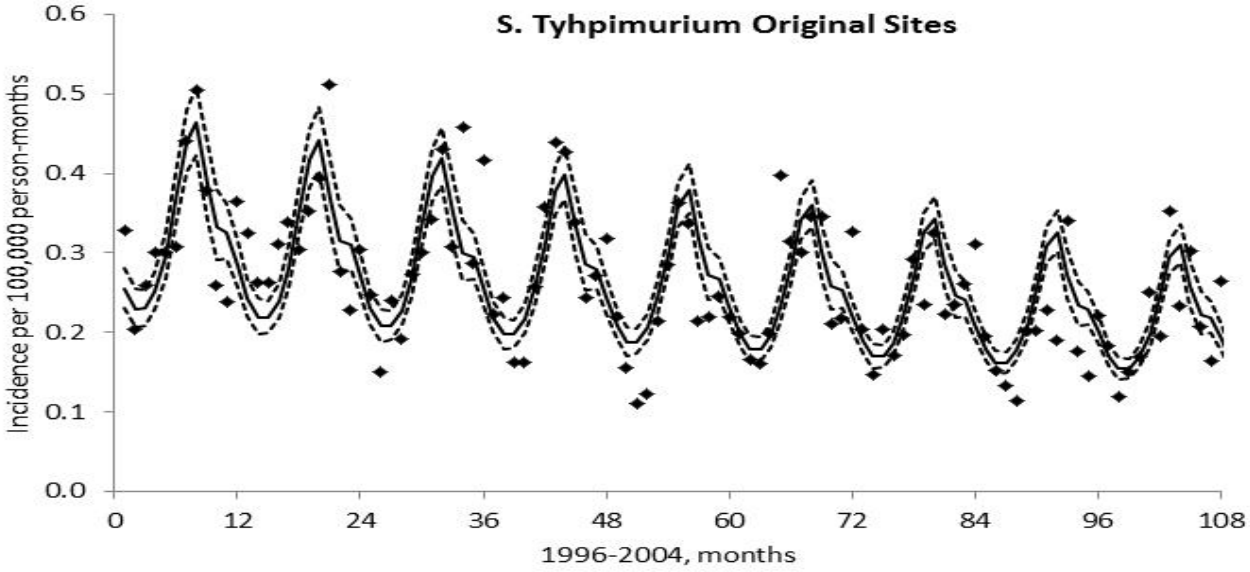

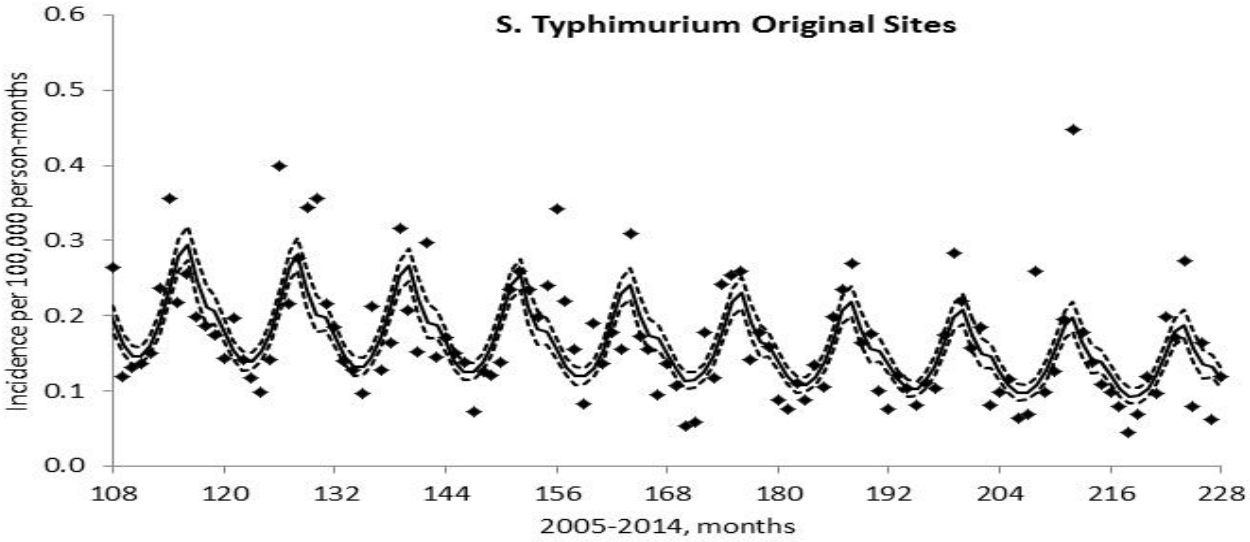

Powell: P-Spline Regression

Limitations

- Serotype results are preliminary, work in progress
- Less smoothness is imposed at domain boundaries
- Reported illness is a proxy, not true incidence
- Not all FoodNet reported illness is foodborne
- Descriptive model, not infer causes
- Uncertainty about generalizing from FoodNet population to national level not quantified

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Disclaimers

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