Optimal Food Safety Sampling Under a Budget Constraint

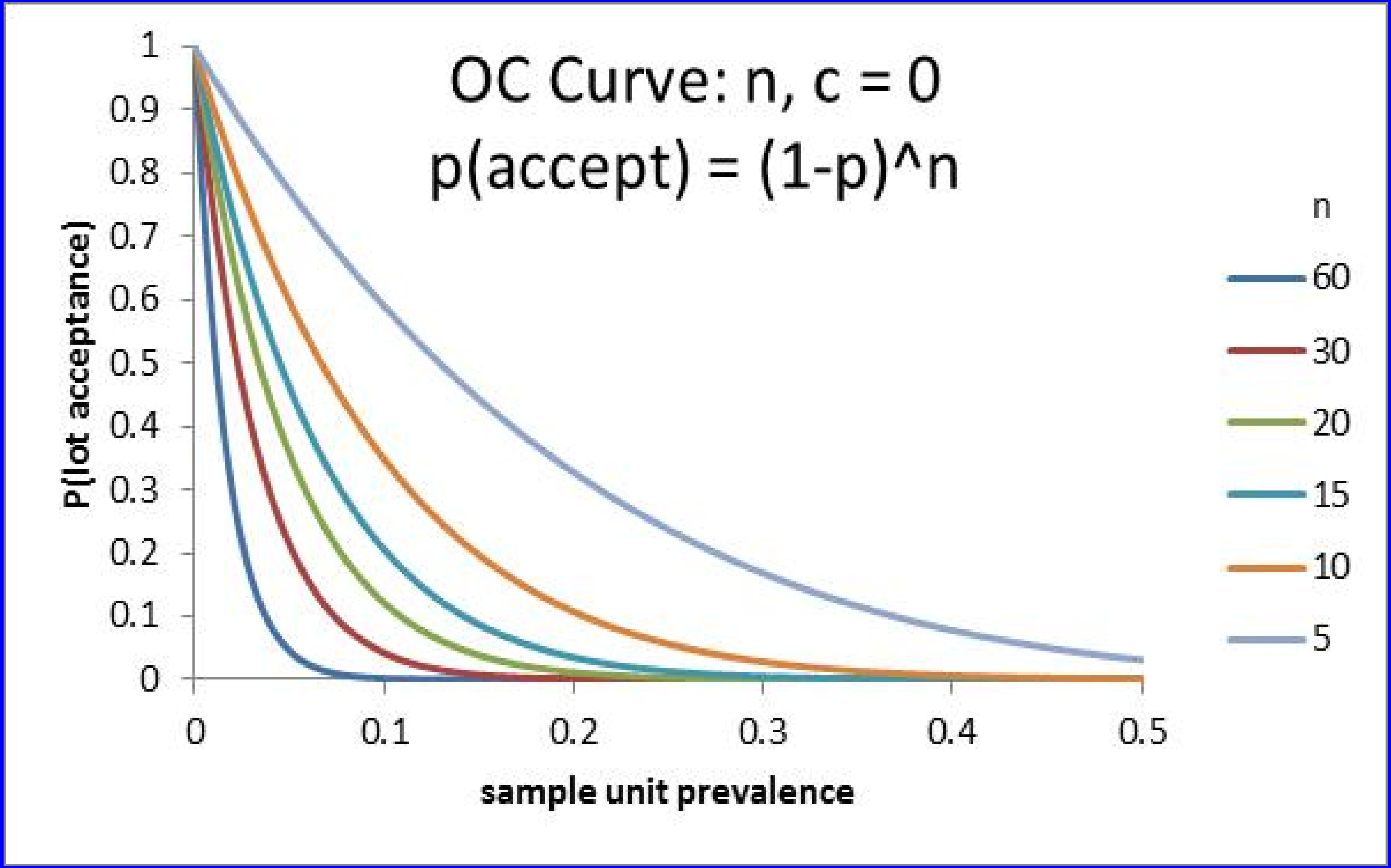
Mark Powell
U.S. Department of Agriculture, Office of Risk Assessment and Cost-Benefit Analysis
Washington, DC

USDA/ORACBA Risk Forum

September 18, 2013

Washington, DC

Conventional Food Safety Sampling Plan Design



Optimization Model for Fixed Prevalence

- $Max\ L_R = m[1 - q^n]$
  - L_R = contaminated lots rejected
  - m = lots
  - n = samples per lot
  - $q = (1 - \text{p})$
  - p = sample unit prevalence
  - $1\text{-}q^n$ = p(reject lot)
- *S.t.:* Budget constraint (C_T)
  - $C_T \geq m(C_l + nC_n)$

Optimization Model

- $Max\ L_R = \frac{C_T}{C_l + nCn}[1 - q^n]$
  - $m = \frac{C_T}{C_l + nC_n}$ (budget constraint)
  - C_T = budgeted total sampling cost ($)
  - C_l = cost per lot ($)
  - C_n = cost per sample ($)

$$n_{opt}(C_T, C_l, C_n, p) \rightarrow \frac{\partial(L_R)}{\partial(n)} = \frac{[C_l + nC_n][-C_T q^n ln(q)] - [C_T(1-q^n)]C_n}{[C_l + nC_n]^2} = 0$$

Optimization Model

- Obj Fxn: $L_R = m(1 - q^n) = f(m, n|q)$
- Constraint: $C_T \geq m(C_l + nC_n)$
- $L = f(m, n|q) + \lambda[C_T - m(C_l + nC_n)]$

Optimization Model

1) $\frac{\partial L}{\partial m} = \frac{\partial f}{\partial m} - \lambda(C_l + nC_n) = 0$

2) $\frac{\partial L}{\partial n} = \frac{\partial f}{\partial n} - \lambda m C_n = 0$

3) $\frac{\frac{\partial f}{\partial m}}{\frac{\partial f}{\partial n}} = \frac{C_l + nC_n}{mC_n}$

4) $n = \frac{\frac{\partial f}{\partial m}}{\frac{\partial f}{\partial n}} m - \frac{C_l}{C_n}$

5) $\frac{\frac{\partial f}{\partial m}}{\frac{\partial f}{\partial n}} = \frac{(1-q^n)}{-mq^n ln(q)}$

6) $n + \frac{(1-q^n)}{q^n ln(q)} + \frac{C_l}{C_n} = 0$

Note: $n_{opt} = f\left(p, \frac{C_l}{C_n}\right)$

Sampling and Testing Assumptions

- Test sensitivity ignored, specificity assumed 100%
- For presence-absence testing:
  - sample unit prevalence must be referenced to a sample unit size
- For quantitative limits:
  - $\hat{\mu}\ (cfu/g) = \frac{\lambda\ (cfu_{obs})}{s_{test}},\ Var(\hat{\mu}) = \frac{\lambda}{(s_{test})^2}$
  - assume negligible measurement error

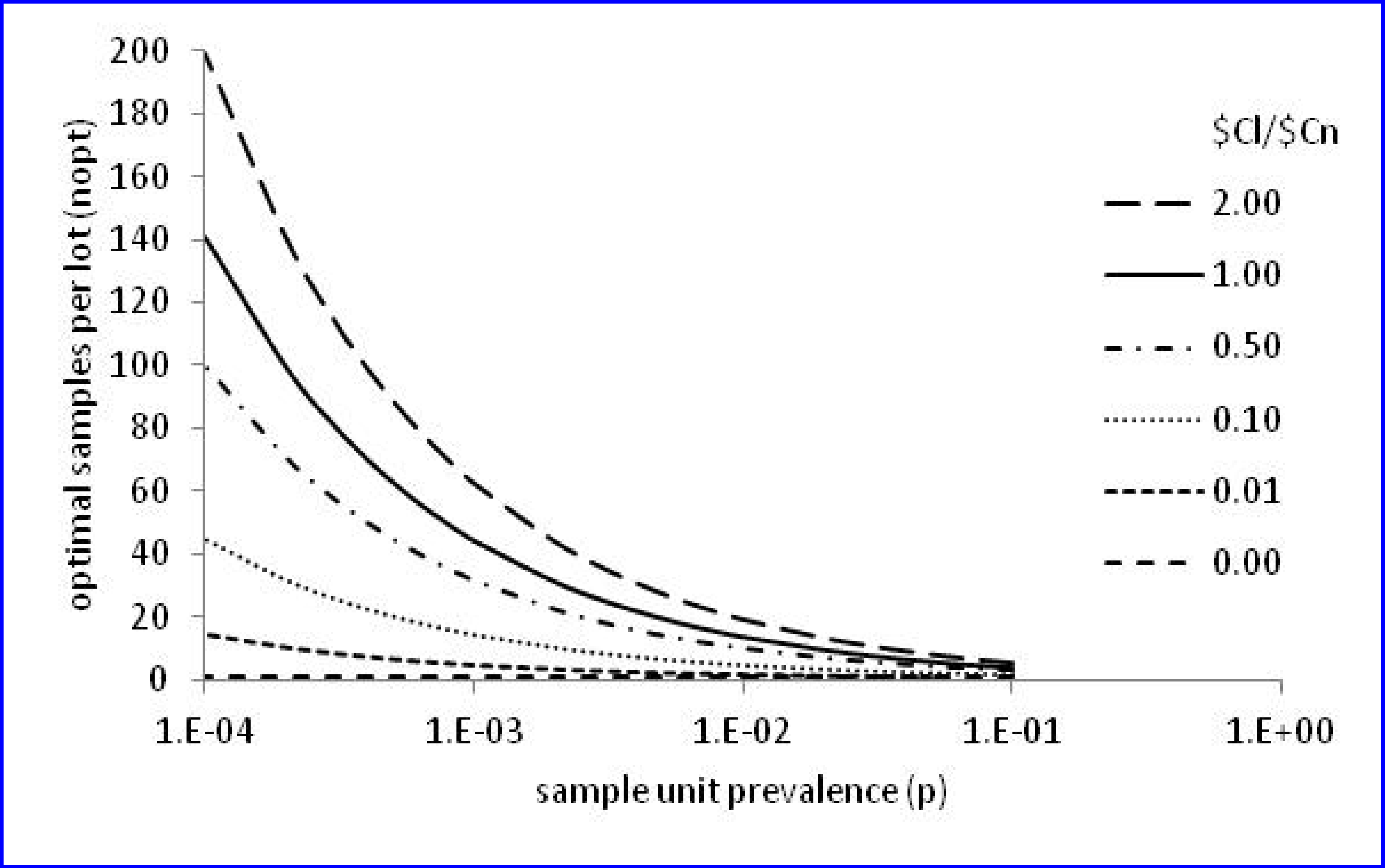
Cost Assumptions

- \$Cost of testing a lot = $\$C_l + n\$C_n$
- Assume $\$C_l/\$C_n$ ranges from 0 to 2

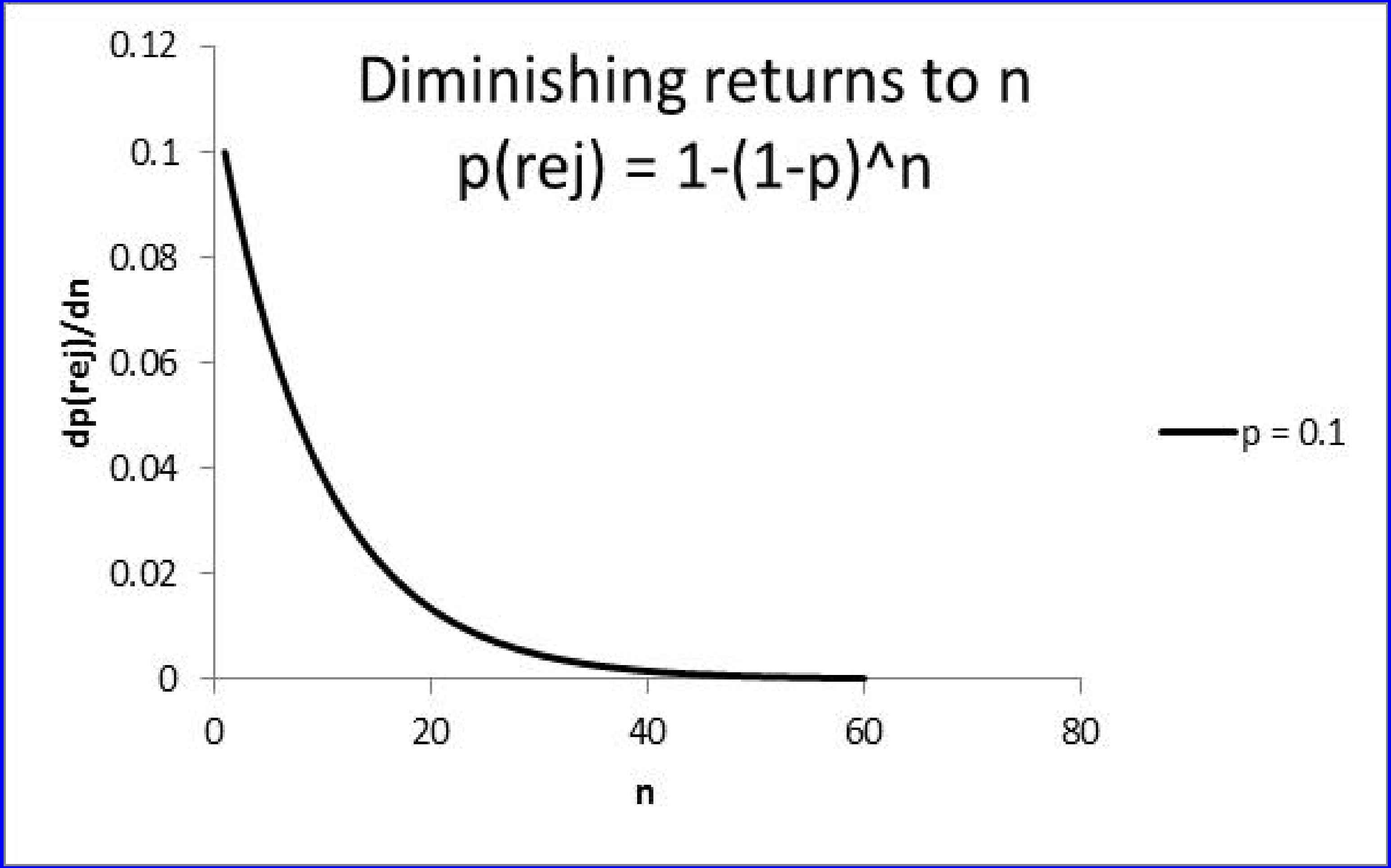
Results

- If budget constraint does not permit testing 100% of lots, n_{opt} for a given sample unit prevalence (*p*) depends only on the cost ratio (C_l/Cn).
- The budget constraint (C_T) determines absolute number of lots tested in a budget period (*m*) or the frequency of lot inspection (1/*m*)

Results

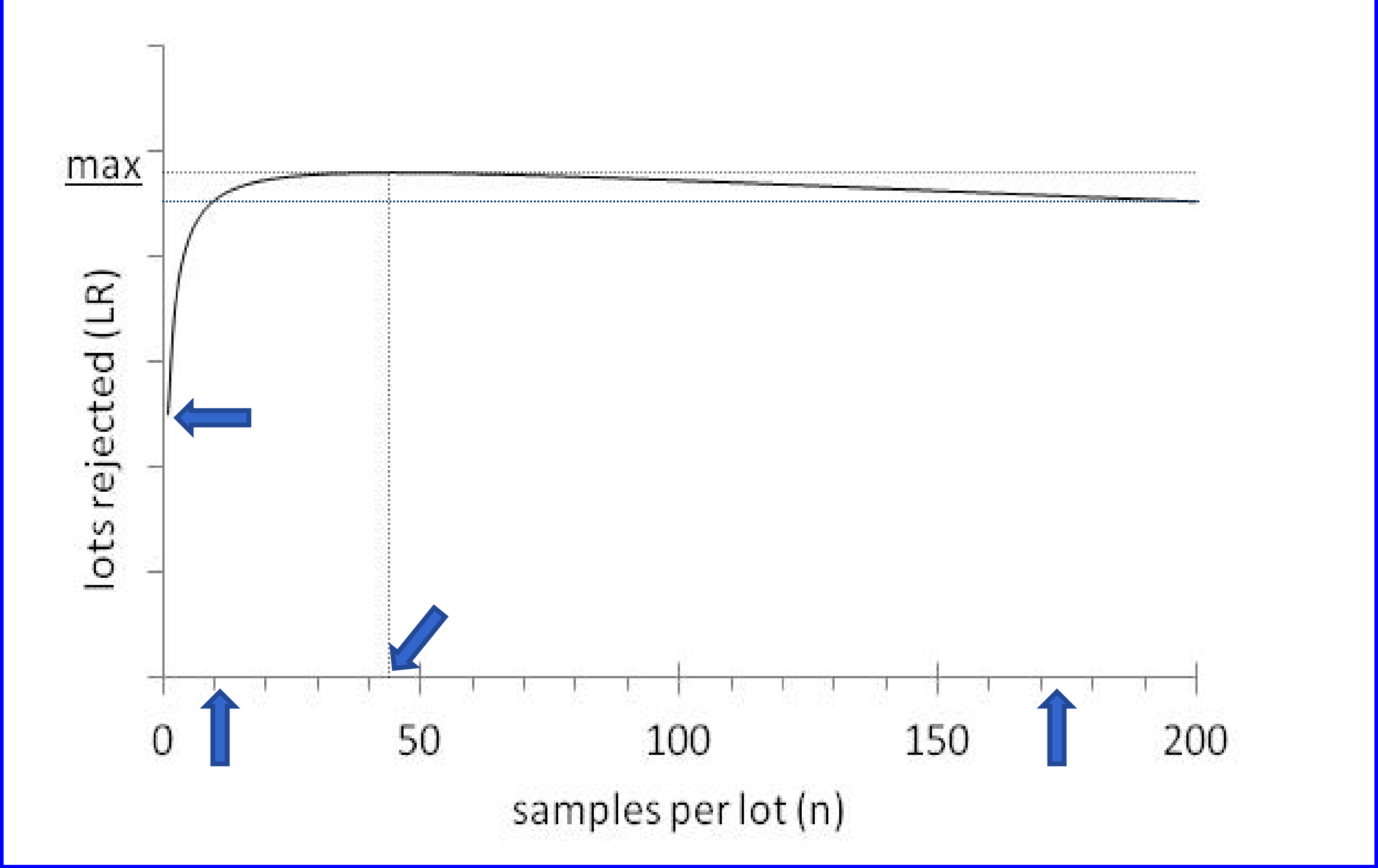

optimal samples per lot (nopt)
200
180
160
140
120
100
80
60
40
20
0
1.E-04
1.E-03
1.E-02
1.E-01
1.E+00
sample unit prevalence (p)
$Cl/$Cn
2.00
1.00
0.50
0.10
0.01
0.00

Results


Diminishing returns to n
p(rej) = 1-(1-p)^n
0.12
0.1
0.08
0.06
0.04
0.02
0
dp(rej)/dn
0
20
40
60
80
n
p = 0.1

Results

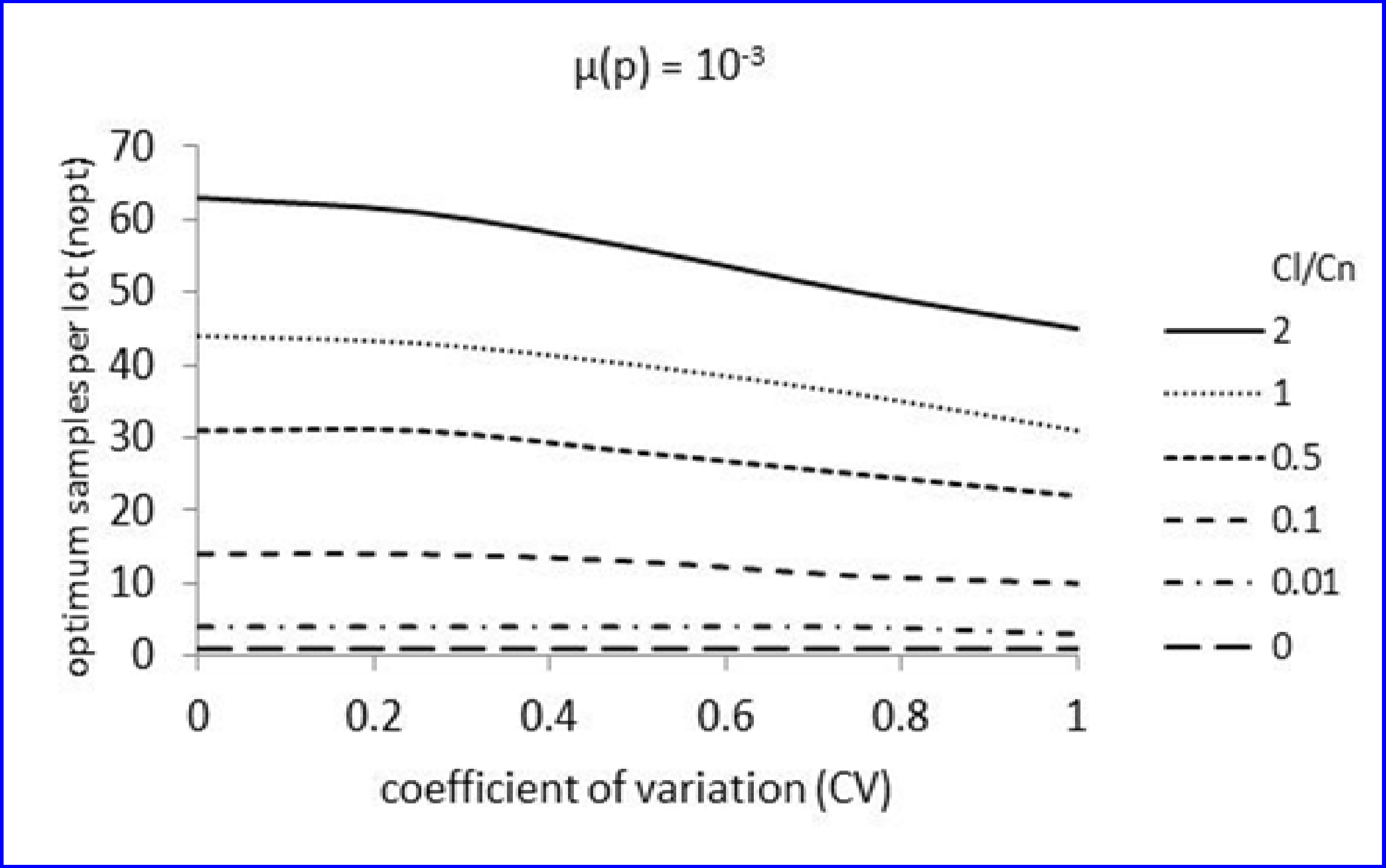
$\$C_l/\$C_n = 1$ and $p = 10^{-3}$



Optimization Model for Variable Prevalence

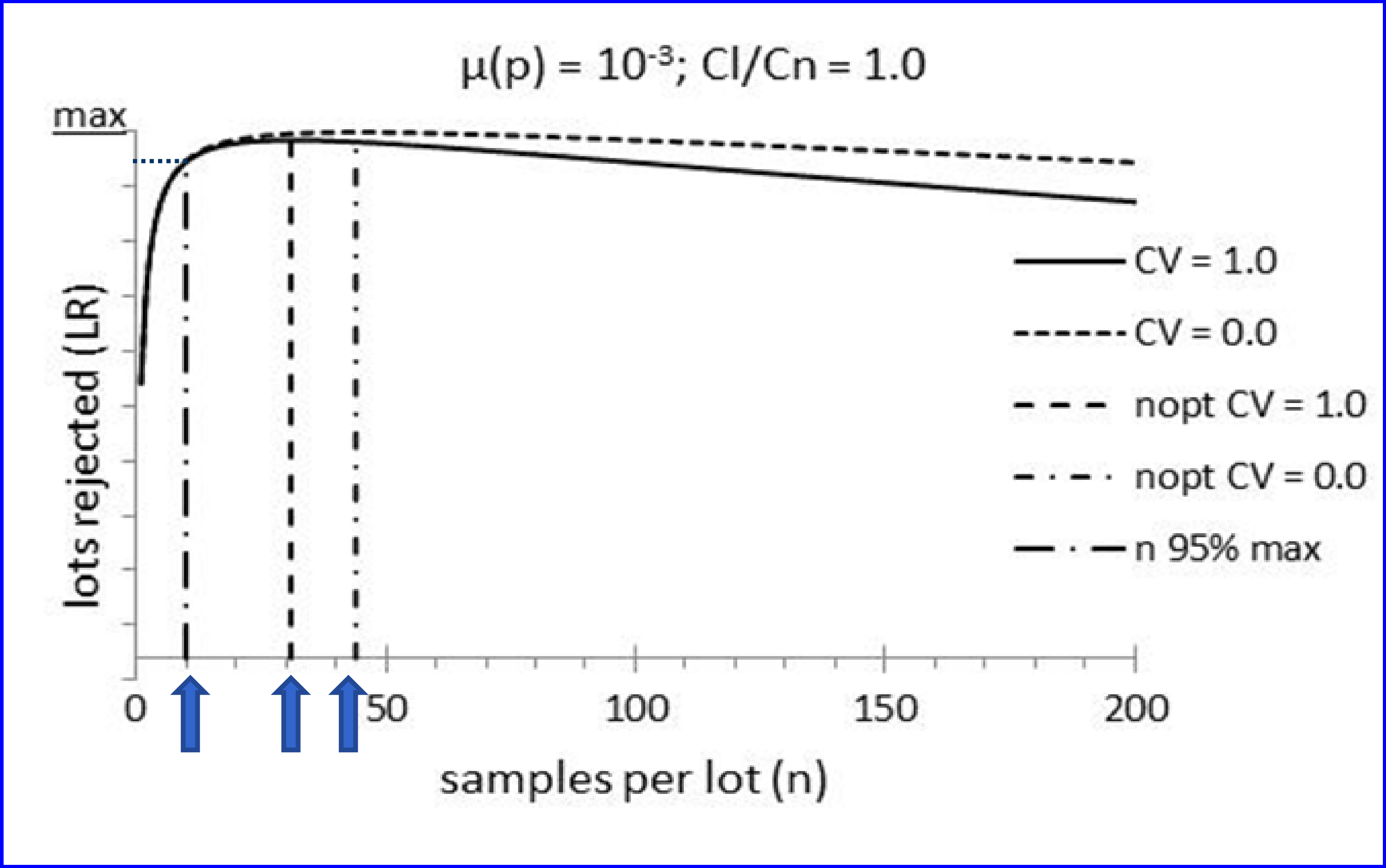
- $E[L_R] = m \int_0^1 (1 - q^n) g\left(p | \mu_p, \sigma_p^2\right) dp$
- $g\left(p | \mu_p, \sigma_p^2\right) = Beta\left(\mu_p, \sigma_p^2\right)$
- $\mu_p = 10^{-1}$ to 10^{-4}
- coeff of var'n ($cv = \sigma_p / \mu_p$) = 0 to 1
- max $E[L_R]$
- s.t. $m = C_T / [C_l + nC_n]$

Results



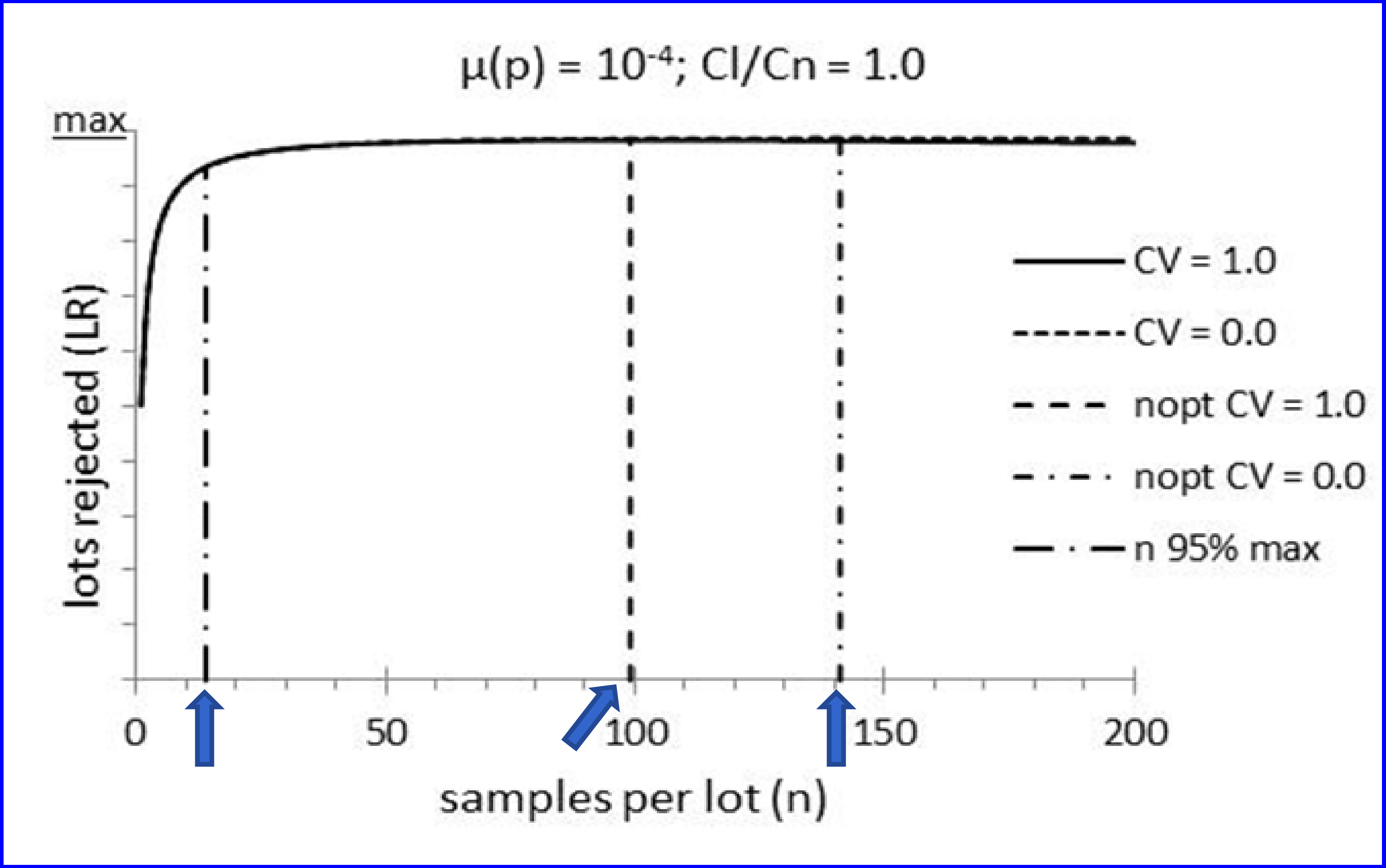
μ(p) = 10-3
optimum samples per lot (nopt)
70
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
1
coefficient of variation (CV)
Cl/Cn
2
1
0.5
0.1
0.01
0

Results



μ(p) = 10-3; Cl/Cn = 1.0
max
lots rejected (LR)
0
50
100
150
200
samples per lot (n)
CV = 1.0
CV = 0.0
nopt CV = 1.0
nopt CV = 0.0
n 95% max

Results



μ(p) = 10-4; Cl/Cn = 1.0
max
lots rejected (LR)
CV = 1.0
CV = 0.0
nopt CV = 1.0
nopt CV = 0.0
n 95% max
0
50
100
150
200
samples per lot (n)

Conclusion

- National Research Council (1985): Food safety sampling plans based on “sound statistical concepts” need to “achieve a high degree of confidence in the acceptability of a lot.”
- Economic design of measures is not new.

Conclusion

- Scarce resources should force us to consider the tradeoff between depth (n) and coverage (m).
- Multiple, competing objectives for sampling.
- What inferences can be drawn from a sampling plan?

Disclaimers

The opinions expressed herein are the views of the author and do not necessarily reflect the official policy or position of the United States Department of Agriculture. Reference herein to any specific commercial products, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government.