Optimal Food Safety Sampling Under a Budget Constraint

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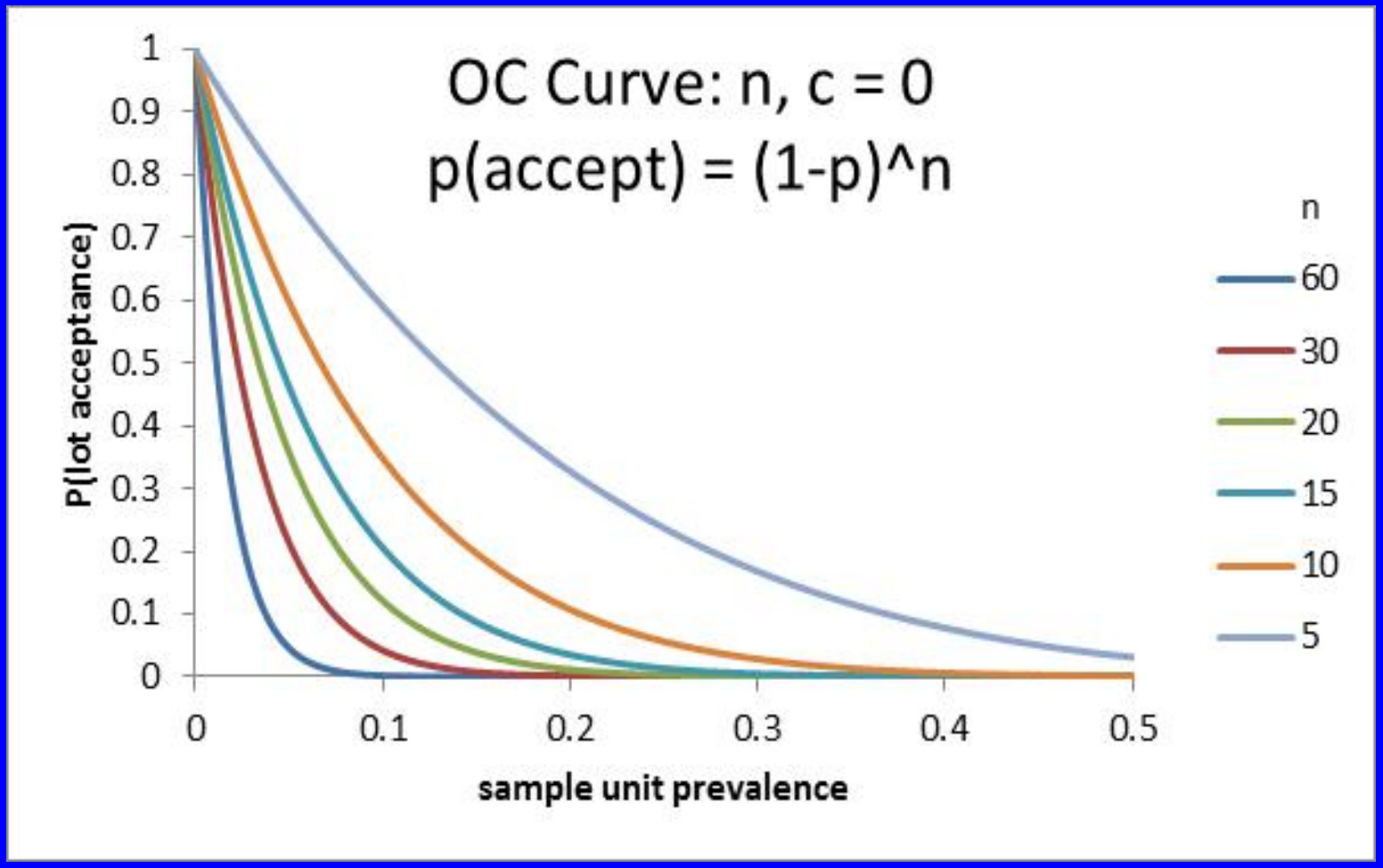
Conventional Food Safety Sampling Plan Design



Optimization Model

- $Max\ L_R = m[1 - q^n]$
  - $L_R =$ contaminated lots rejected
  - $m =$ lots
  - $n =$ samples per lot
  - $q = (1 - \mathrm{p})$
  - $p =$ sample unit prevalence
  - $1\text{-}q^n =$ p(reject lot)
- *S.t.:* Budget constraint (C_T)
  - $C_T \geq m(C_l + nC_n)$

Optimization Model

- $Max\ L_R = \frac{C_T}{C_l + nCn}[1 - q^n]$
  - $m = \frac{C_T}{C_l + nC_n}$ (budget constraint)
  - $C_T =$ budgeted total sampling cost ($)
  - $C_l =$ cost per lot ($)
  - $C_n =$ cost per sample ($)

$$n_{opt}(C_T, C_l, C_n, p) \rightarrow \frac{\partial(L_R)}{\partial(n)} = \frac{[C_l + nC_n][-C_T q^n ln(q)] - [C_T(1 - q^n)]C_n}{[C_l + nC_n]^2} = 0$$

Optimization Model

- Obj Fxn: $L_R = m(1 - q^n) = f(m, n|q)$
- Constraint: $C_T \geq m(C_l + nC_n)$
- $L = f(m, n|q) + \lambda[C_T - m(C_l + nC_n)]$

Optimization Model

1) $\frac{\partial L}{\partial m} = \frac{\partial f}{\partial m} - \lambda(C_l + nC_n) = 0$

2) $\frac{\partial L}{\partial n} = \frac{\partial f}{\partial n} - \lambda m C_n = 0$

3) $\frac{\frac{\partial f}{\partial m}}{\frac{\partial f}{\partial n}} = \frac{C_l + nC_n}{mC_n}$

4) $n = \frac{\frac{\partial f}{\partial m}}{\frac{\partial f}{\partial n}} m - \frac{C_l}{C_n}$

5) $\frac{\frac{\partial f}{\partial m}}{\frac{\partial f}{\partial n}} = \frac{(1-q^n)}{-mq^n ln(q)}$

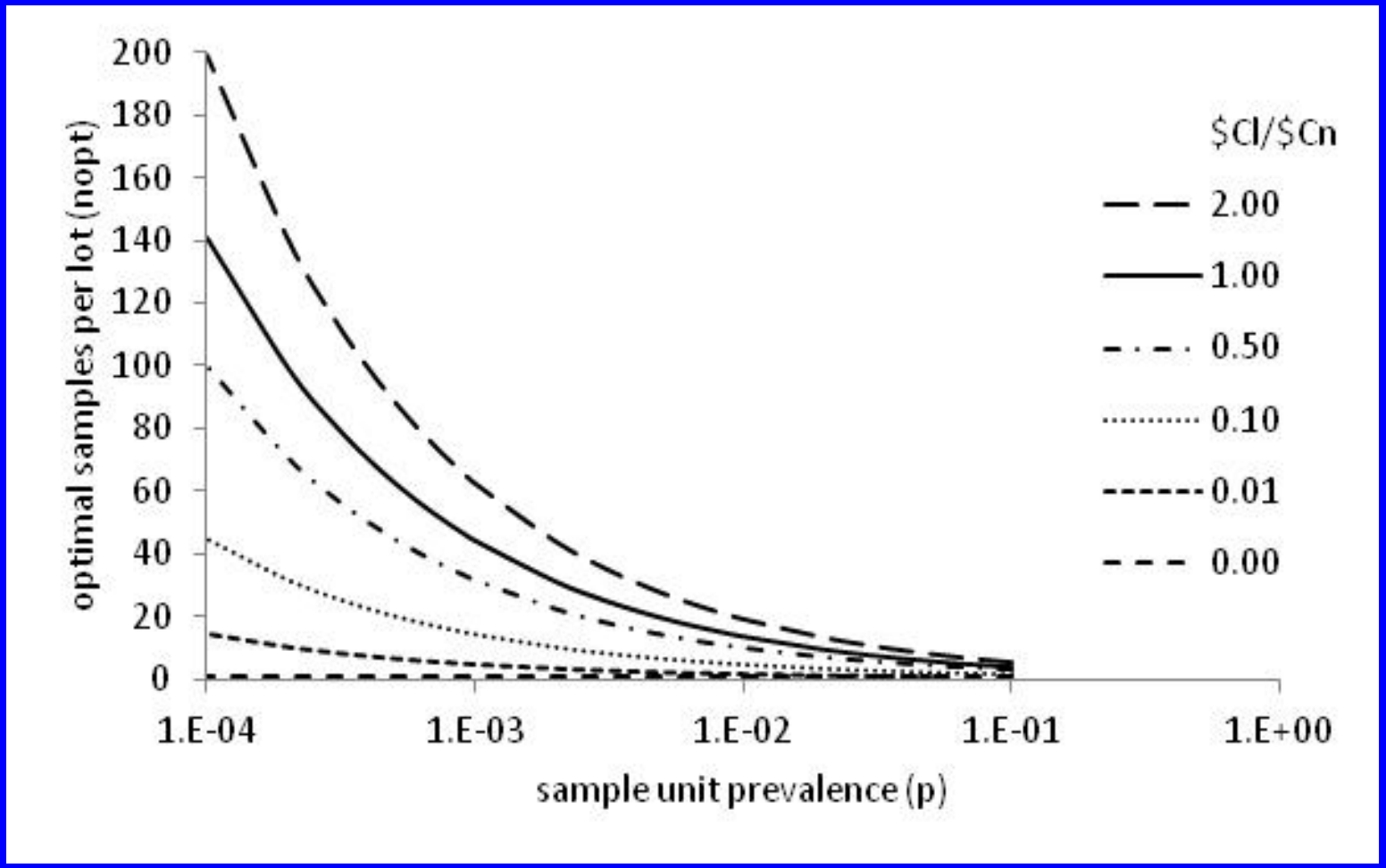
6) $n + \frac{(1-q^n)}{q^n ln(q)} + \frac{C_l}{C_n} = 0$

Note: $n_{opt} = f\left(p, \frac{C_l}{C_n}\right)$

Results

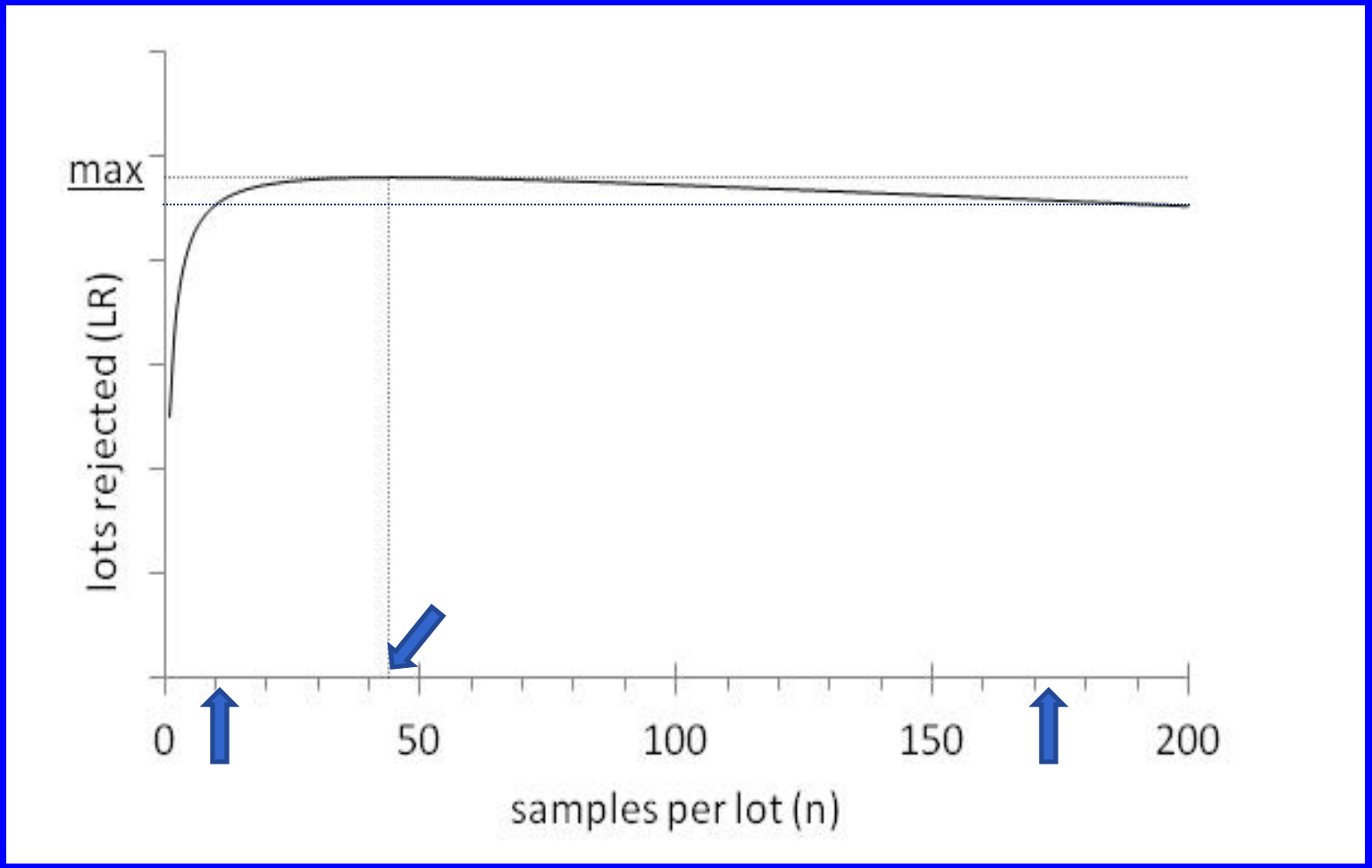
- If budget constraint does not permit testing 100% of lots, n_{opt} for a given sample unit prevalence (p) depends only on the cost ratio (C_l/Cn).
- The budget constraint (C_T) determines absolute number of lots tested in a budget period (m) or the frequency of lot inspection ($1/m$)

Results


200
180
160
140
120
100
80
60
40
20
0
optimal samples per lot (nopt)
1.E-04
1.E-03
1.E-02
1.E-01
1.E+00
sample unit prevalence (p)
$Cl/$Cn
2.00
1.00
0.50
0.10
0.01
0.00

Results

$\$C_l/\$C_n = 1$ and $p = 10^{-3}$



Conclusion

- National Research Council (1985): Food safety sampling plans based on “sound statistical concepts” need to “achieve a high degree of confidence in the acceptability of a lot.”
- Economic design of measures is not new.
- Scarce resources should force us to consider the tradeoff between depth (n) and coverage (m).
- Multiple, competing objectives for sampling.

Disclaimers

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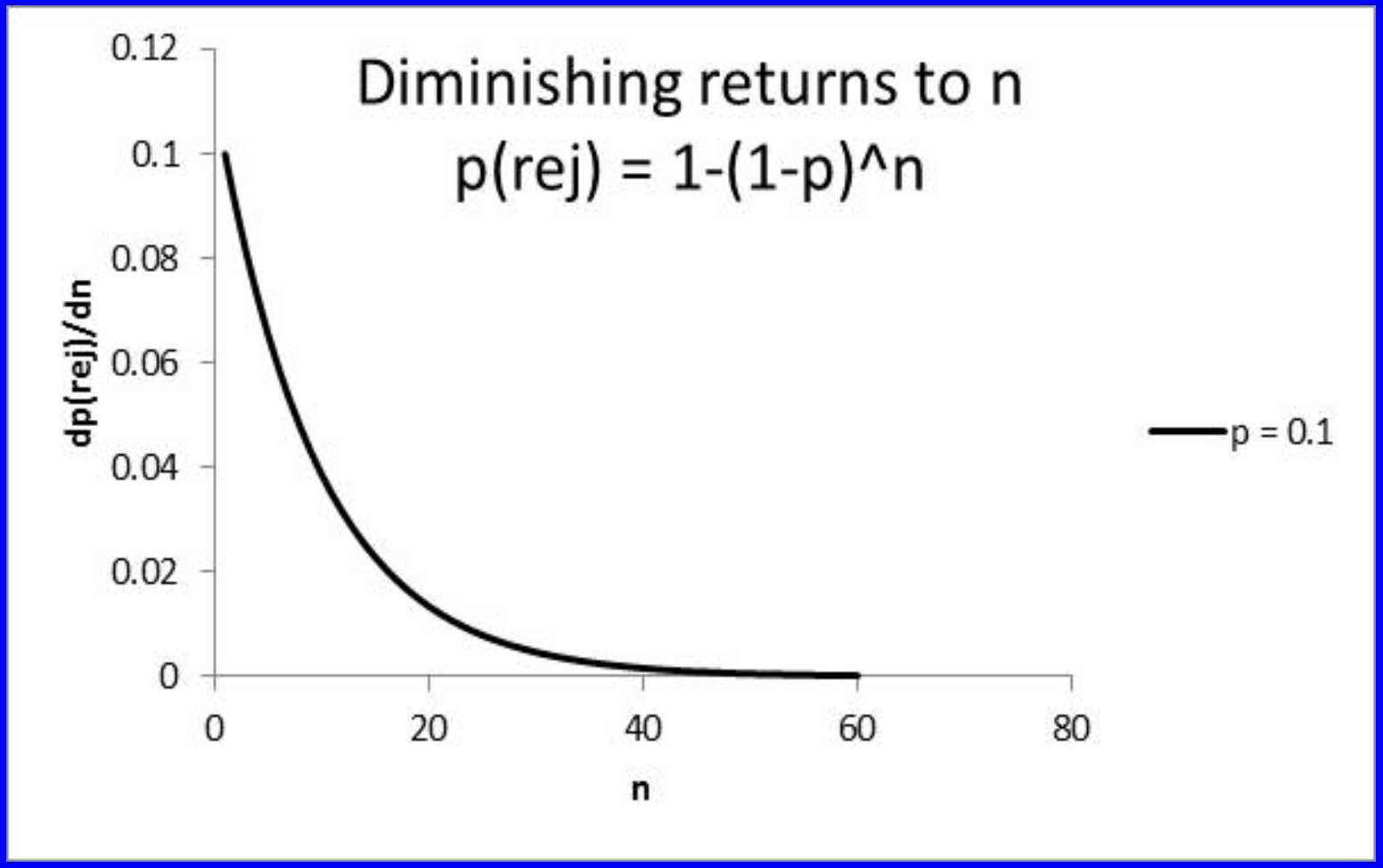
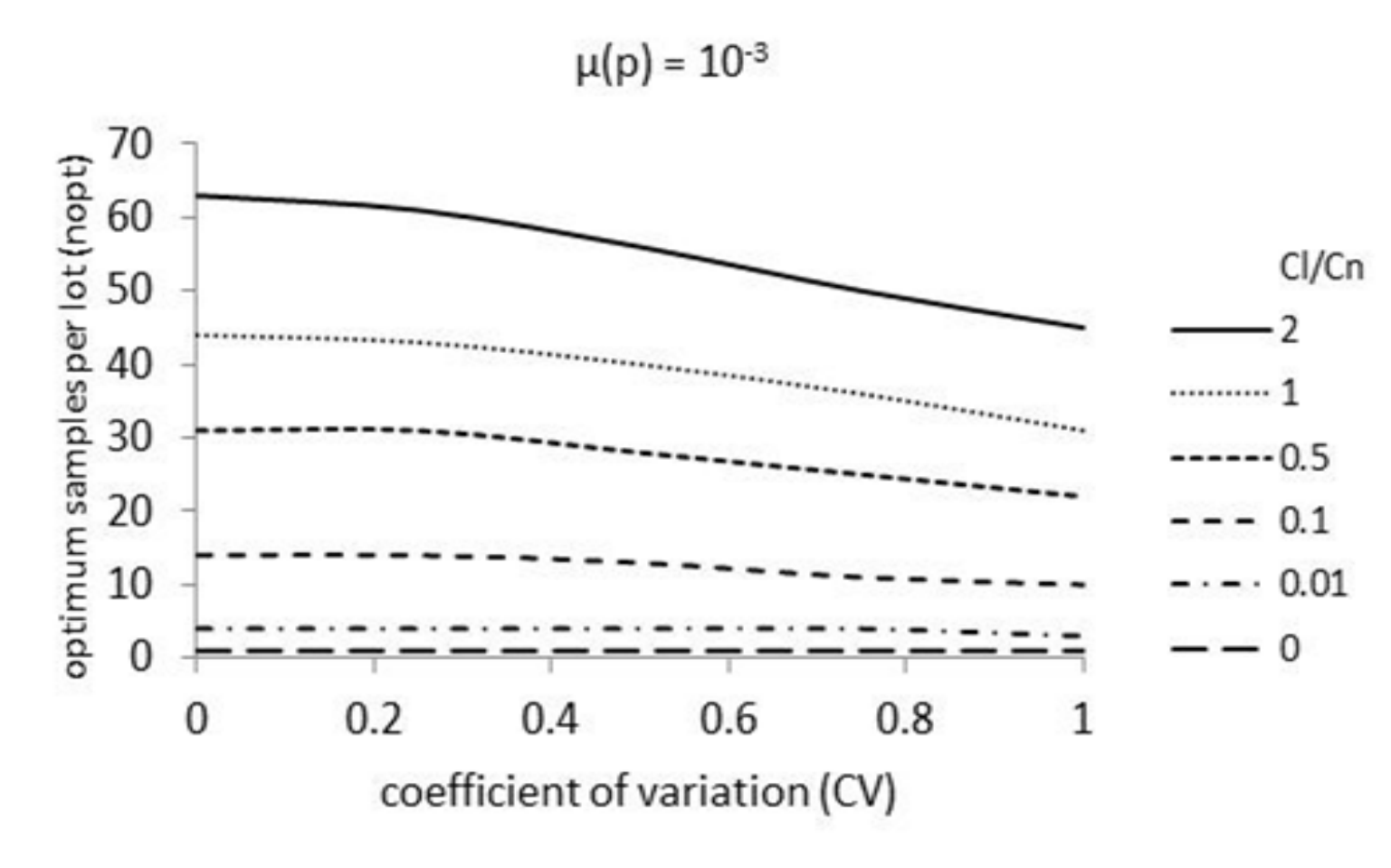
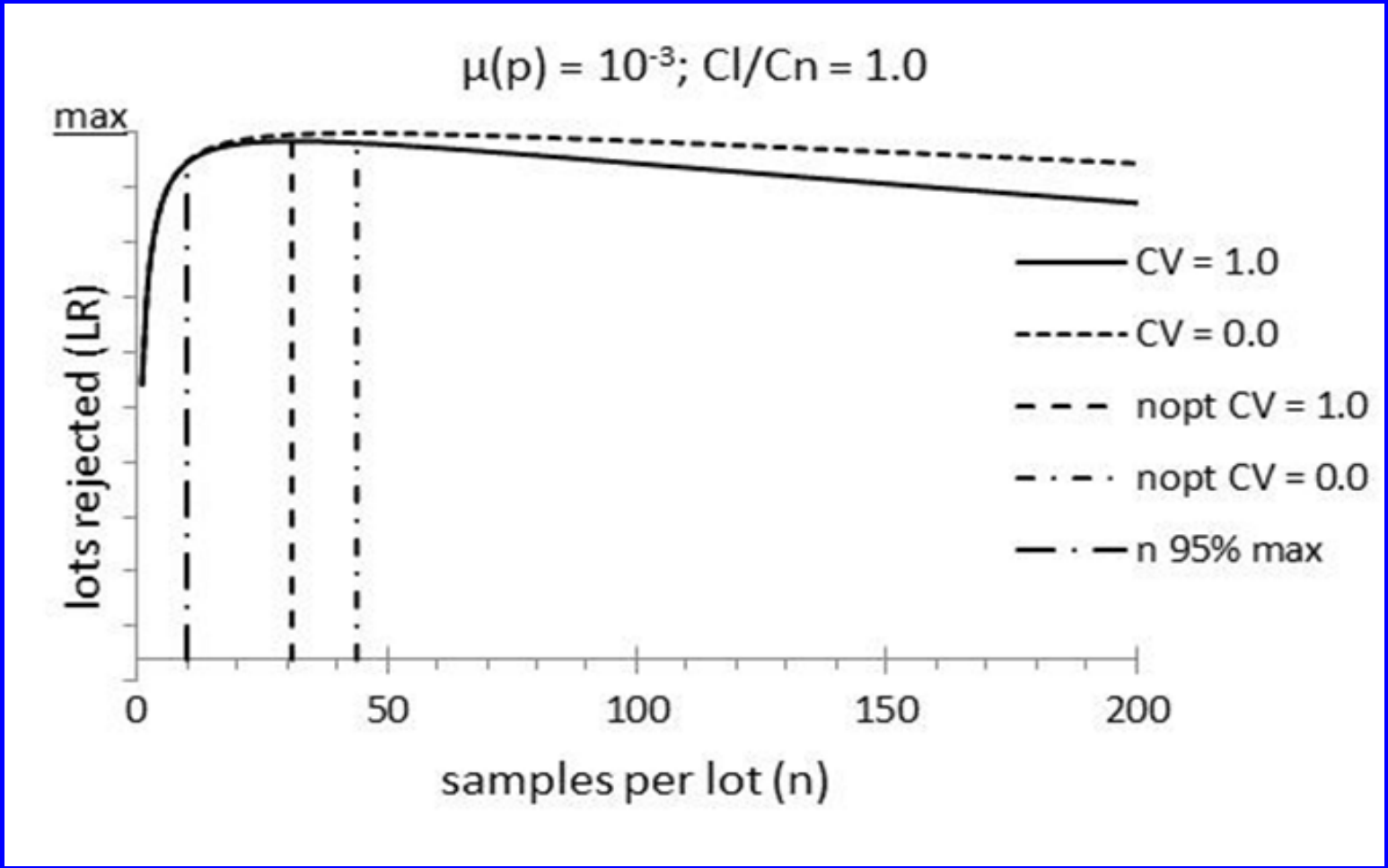
Results


Diminishing returns to n
p(rej) = 1-(1-p)^n
dp(rej)/dn
0.12
0.1
0.08
0.06
0.04
0.02
0
0
20
40
60
80
n
p = 0.1

Results


μ(p) = 10-3
optimum samples per lot (nopt)
70
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
1
coefficient of variation (CV)
Cl/Cn
2
1
0.5
0.1
0.01
0

Results


μ(p) = 10-3; Cl/Cn = 1.0
max
lots rejected (LR)
0
50
100
150
200
samples per lot (n)
CV = 1.0
CV = 0.0
nopt CV = 1.0
nopt CV = 0.0
n 95% max